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USE OF THE SCATTERED LIGHT METHOD IN ORDER TO DETERMINE THE STRESS INTENSITY FACTOR K_{III} IN THREE-DIMENSIONAL PROBLEMS

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Strain "freezing" [1, 2] and scattered light [3] methods are used for experimental determination of the stress intensity factor (SIF) K_{III} in studying solid structural elements with surface or internal cracks. The scattered light method exhibits considerable potential and marked advantages over the "freezing" method by making it possible to obtain the required data without cutting up the model. However, this method has not been used extensively due to the complexity of experiments and interpretation of measured results. For example, in [3] it is suggested that the model is examined in a plane perpendicular to the crack front by a light beam intersecting the tip of the crack. This illumination scheme requires careful selection of the immersion liquid and treatment of the crack edge surfaces, and also rotation of the model or the device around the point of intersection of the crack front by the beam.

A simpler procedure is described in the present work for carrying out an experiment which makes it possible to carry over methods known in plane photoelasticity for treating experimental data for determining the SIF to the case of determining K_{III} for spatial cracks.

For longitudinal shear stresses close to the tip of a crack are expressed as follows:

$$\begin{gathered}\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = 0,\\ \tau_{xz} = K_{III}(2\pi r)^{-1/2}\sin(\theta/2), \quad \tau_{yz} = K_{III}(2\pi r)^{-1/2}\cos(\theta/2),\end{gathered} \tag{1}$$

where x, y, z is an orthogonal coordinate system orientated so that axis z is tangential to the crack front at point O (Fig. 1); r, θ are polar coordinates.

Since the value of optical difference for the path of the light beam due to the difference in quasiprincipal stresses which operate in a plane perpendicular to the illuminating beam is measured by the scattered light method, then it will be most effective to examine it in plane xOy parallel to axis x. It is important that the direction of principal stresses

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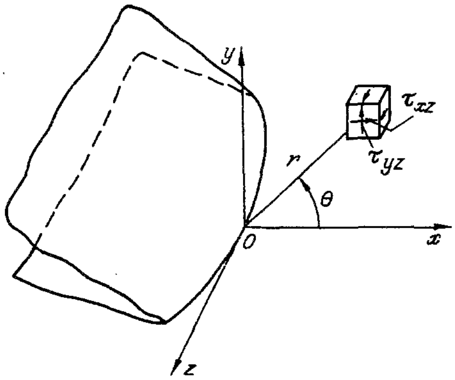

Fig. 1

does not change its orientation along these directions (absence of rotation for the principal axes). In this case the order of the interference band m_{ij} for point x_i taken in beam $y = y_j$ is connected with stresses by the relationship

$$m_{ij} = 2C \int_{x_0}^{x_i} \tau_{yz}\, dx \quad (2)$$

(x_0 is the coordinate of the point of entry of the beam of light ($y = y_j$) into the model, C is an opticomechanical constant of the model material).

The direction of principal stresses is orientated the same at all points of plane xOy so that it is possible to record the picture of interference bands with examination of this plane by a blade of polarized light directed along axis x. In this way the beam of light is arranged above or below the crack and it does not intersect its front. By selecting n points in straight lines parallel to axis x (y = const) with known values of the order of interference bands and bearing in mind relationships (1) and (2) we have

$$K_{\mathrm{III}} = \frac{1}{n} \sum_{j=1}^{k} \sum_{i=1}^{l_j} \frac{\Delta m_{ij}}{\int_0^{x_i} f(x, y_j)\, dx}, \quad (3)$$

where k and l_j are the numbers of straight lines y = const and points in them taken in the calculation; $n = \sum_{j=1}^{k} l_j$; $\Delta m_{ij} = m_{0j} - m_{ij}$ (m_{0j} is order of bands in straight line y_j with $x = 0$); $f(x, y_j) = C \left[\frac{y_j + (x^2 + y_j^2)^{1/2}}{2\pi (x^2 + y_j^2)}\right]^{1/2}$. Equation (3) assumes determination of K_{III} by a direct simple method for treating experimental information. However, the possibility of obtaining a large number of experimental data Δm_{ij} from one picture of bands makes it also possible to use more complex treatment methods.

The method described which is based on solution (1) is valid in a limited zone at the crack tip. As a rule in order to prepare a crack model it is simulated by a notch [3] (surface) or cavity with an antiadhesive insert [5]) (internal). In either case the radius of curvature ρ of the "crack" tip has an entirely specific value which distorts the field of model stresses and leads to marked errors. Consideration of the effect of ρ is possible by using a solution for twisting of an infinite axisymmetrical body with an external hyperbolic cut-out [6]:

$$\tau_{yz} = \frac{3p\left(\sqrt{a/\rho + 1} + 1\right)\sqrt{a/\rho}}{4\left(2\sqrt{a/\rho + 1} + 1\right)\left(\sqrt{a/\rho + 1} - 1\right)} \frac{\sin v \cos v}{(\mathrm{sh}^2 u + \cos^2 v)\,\mathrm{ch}\, u}. \quad (4)$$

Here a is radius of the undamaged part; p is load parameter; u, v are elliptical coordinates. In our case the connection between the coordinate systems is written as

$$x = a(\mathrm{sh}\, u \cos v - 1), \quad y = a\, \mathrm{ch}\, u \sin v.$$

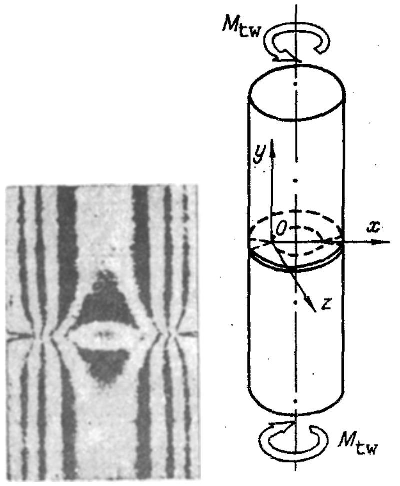

Fig. 2

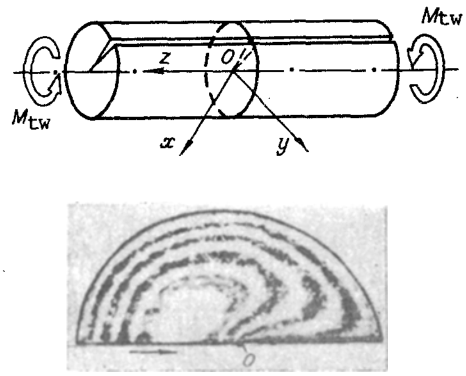

Fig. 3

We select parameters a and p so that stresses in the model and in the body with a hyperbolic cut-out coincide in some region by assuming that in this way the value of ρ for the cut-out is equal to the radius of curvature at the "crack" tip in the model, i.e., for the order of bands measured at n points of plane xOy we minimize the function

$$F(a, p) = \sum_{j=1}^{k} \sum_{i=1}^{l_j} \left[\Delta m_{ij} - 2C \int_0^{x_i} \tau_{yz}(a, p)\, dx \right]^2 \tag{5}$$

(τ_{yz} is determined from (4)).

Thus, for values of a^* and p^* corresponding to the minimum $F(a, p)$, by assuming that a limited changeover with respect to stresses from a body with a hyperbolic cut-out to a body with an external annular crack is similar to a changeover from a model with a crack to a structural element designed with a crack, we have

$$K_{III} = (3/8)p^* \sqrt{\pi a^*}. \tag{6}$$

In order to check the accuracy of this method studies were carried out for the following test problems: twisting a cylinder with an annular crack and a cylinder with a radial crack. Two models were prepared with this aim.

The first model is a cylinder 37.6 mm in diameter with an annular notch 5 mm deep whose radius of curvature at the tip $\rho_1 = 0.05$ mm (Fig. 2). The material of the model is an epoxy compound based on resin ÉD-16 for which $C_1 = 13.5$ kN/m. The second model made of polyurethane SKU-6 is a cylinder 44 mm in diameter with a radial notch 22 mm deep over the whole length of the generating line (Fig. 3). The notch in the specimens was prepared in two stages. At first polyurethane compound was poured followed by polymerization in the form of a cylinder with a radial notch 18 mm deep and 0.5 mm wide, and then a notch of the required depth was made with a sharp blade. In this case $\rho_2 = 0.1$ mm. This preparation method for a crack made it possible to exclude friction over the large surface of its sides with twisting of the model. The opticomechanical constant for SKU-6 material is $C_2 = 0.239$ kN/m.

Studies were carried out in a light scattering device consisting of the following optical elements: a He–Ne–laser LG-75 as a polarized light source; a half-wave plate for rotating the plane of polarized light; two cylindrical lenses and a slit diaphragm which forms a beam of light with a light blade 37 mm wide and up to 0.5 mm thick; a model in an immersion bath fastened to the lower grip of a loading device; a photoreceiver for recording interference pictures. In accordance with the procedure described the model was illuminated by a beam of polarized light in plane xOy (see Figs. 2 and 3). Interference pictures were recorded in direction Oz for the first model and at an angle of 70° to plane xOy for the second. Given in Fig. 2 is the band picture observed in the first model loaded by twisting moment $M_{tw} = 2.4$ N·m and combined for two positions of the light blade: above and below the crack plane. The band picture for the second model is shown in Fig. 3 ($M_{tw} = 0.87$ N·m; the direction of illumination is denoted by an arrow; point O corresponds to the crack tip).

TABLE 1

Model number	M_{tw}, N·m	r, mm	n	K_{III} (N/mm$^{3/2}$)	K_{III}^c (N/mm$^{3/2}$)	δ, %
I	14,4	3,0 (1,5)	25 (10)	8,06 (8,51)	7,81	3,1 (8,8)
	19,2	3,0 (1,5)	25 (10)	10,93 (9,85)	10,43	4,8 (—5,5)
	24,0	3,0 (1,5)	25 (10)	13,40 (13,66)	13,04	3,1 (5,0)
II	0,87	2,0 (1,0)	20 (10)	0,384 (0,346)	0,371	3,5 (—6,7)

With each loading, in order to increase the number of interference bands, the band picture was recorded for whole and half orders. Then in the region of the notch tip 30 points were selected for which coordinates x, y were recorded and values of Δm were determined. Then ten points were selected lying as close as possible to a circle of radius r with a center at the notch tip, and K_{III} was calculated by Eqs. (5), (6), or (3). By measuring the radius of curvature a dependence of K_{III} on r was plotted. The dimensions of the region for the points of which the most reliable information is recorded for the value of SIF sought was equal to the dimensions of the area of curve $K_{III}(r)$ where $K_{III} \simeq$ const. The final value of SIF K_{III} was calculated taking account of all of the measurements in this region.

Results are presented in Table 1. Given here in brackets are values relating to K_{III} calculated by Eq. (3). Theoretical values of SIF K_{III}^c were obtained by equations taken from [7] (for the first model) and [4] (for the second). As can be seen from Table 1, consideration of the radius of curvature at the notch tip makes it possible to increase the size of the measurement region by a factor of about two. As a result of this the accuracy of recording coordinates is improved and there is an increase in the number of measurement points which in the final analysis leads to an increase in the accuracy for determining SIF K_{III}.

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